Technical Article

The Howland Current Pump

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The Howland current pump, shown in Figure 1a, is a circuit that accepts an input voltage v_I, converts it to an output current $i_O = Av_I$, with A as the transconductance gain, and pumps i_O to a load LD, regardless of the voltage v_L developed by the load itself. To see how it works, label it as in Figure 1b, and apply Kirchoff's Current Law and Ohm’s Law.

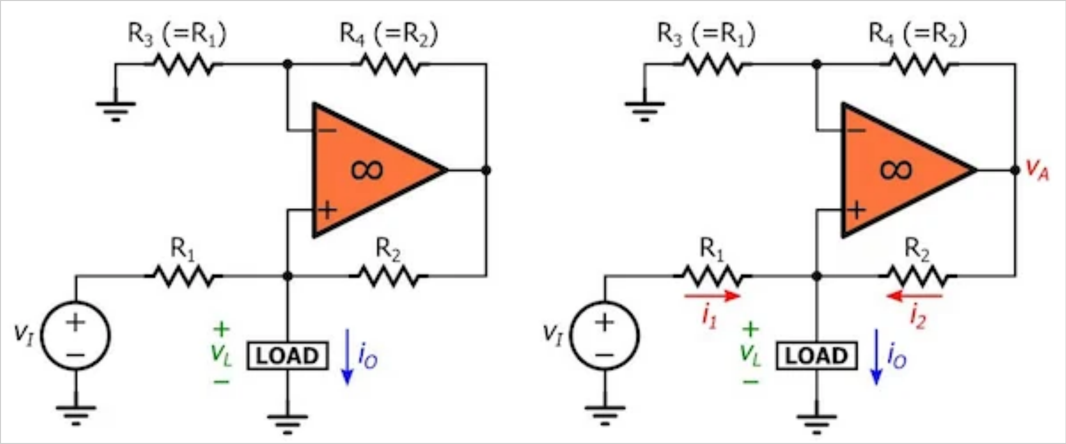


Figure 1. (a) The Howland pump. (b) Properly labeling the circuit for its analysis.

$$i_O = i_1 + i_2 = \frac{v_I - v_L}{R_1} + \frac{v_A - v_L}{R_2}$$

Equation 1

The op-amp, together with R_3 and R_4, forms a non-inverting amplifier with respect to v_L, thus giving

$$v_A = \left(1 + R_4 / R_3\right) v_L$$

Equation 2

Substituting v_A into Equation 1 and collecting, we put i_O into the insightful form

$$i_O = A v_I - \frac{v_L}{R_o}$$

Equation 3

where A is the transconductance gain, in A/V,

$$A = \frac{1}{R_1}$$

Equation 4

and where R_o is the output resistance presented by the circuit to the load,

$$R_o = \frac{R_2}{R_2 / R_1 - R_4 / R_3}$$

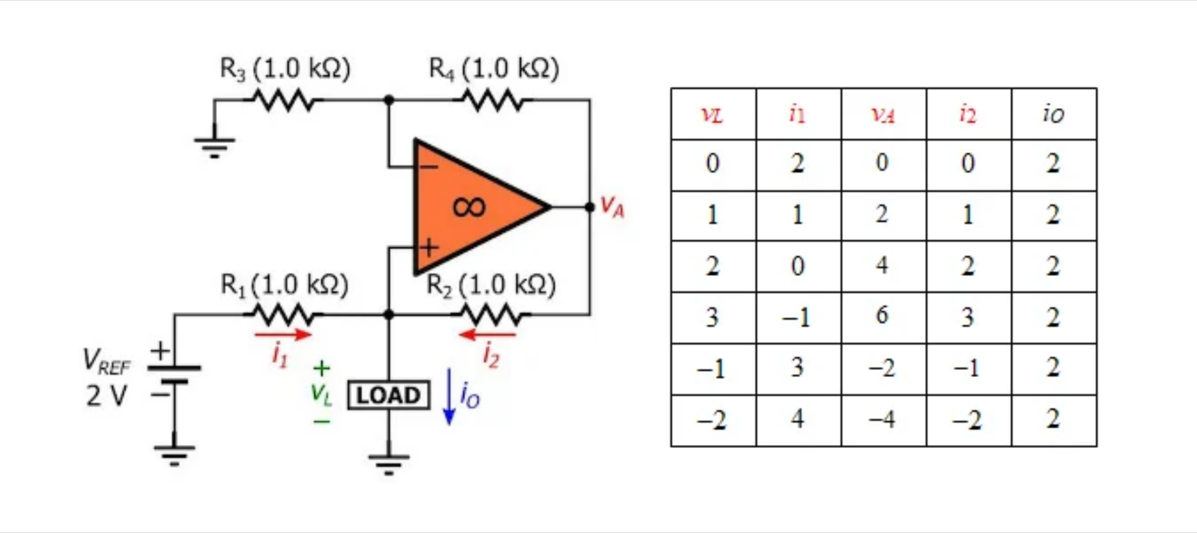
Equation 5

To make i_O independent of v_L we must impose $R_o \rightarrow \infty$, or the balanced-bridge condition.

$$\frac{R_4}{R_3} = \frac{R_2}{R_1}$$

Equation 6

Take a look at the example in Figure 2 and observe, row-by-row, how the op-amp adjusts i_2, via v_A, so as to ensure the same current i_O regardless of v_L.



vL	i1	vA	i2	iO
0	2	0	0	2
1	1	2	1	2
2	0	4	2	2
3	−1	6	3	2
−1	3	−2	−1	2
−2	4	−4	−2	2

Figure 2. (a) A 2 mA current source, and (b) its inner workings for different values of vL (voltages in volts, currents in milliamps; a negative current value means that current flows in the direction opposite to the arrow).

With the polarity of V_{REF} as shown, the pump sources i_O to the load. Inverting the polarity of V_{REF} will cause the pump to sink i_O from the load. Note that for the pump to work properly v_A must always be confined within the linear range of op-amp operation. If the op-amp is driven into saturation, the pump will cease to operate properly.

The Effect of Resistance Mismatches

A practical bridge is likely to be unbalanced because of resistance tolerances, so R_o is likely to be less than infinity. Denoting the tolerances of the resistances in use by p, we note that the denominator D of Equation 5 is maximized when R_2 and R_3 are maximized and R_1 and R_4 are minimized. For $p << 1$, we write

$$D_{\max} = \frac{R_2(1+p)}{R_1(1-p)} - \frac{R_4(1-p)}{R_3(1+p)} \cong \frac{R_2}{R_1}(1+p)^2 - \frac{R_4}{R_3}(1-p)^2 \cong \frac{R_2}{R_1}\left[(1+2p)-(1-2p)\right] \cong \frac{R_2}{R_1}4p$$

Here we have incorporated the relationship of Equation 6, applied approximation

$$1/(1 \mp p) \cong 1 \pm p$$

and ignored quadratic terms in p. Substituting into Equation 5 gives

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$$R_{o(\min)} = \frac{R_2}{D_{\max}} \cong \frac{R_1}{4p}$$

Equation 7

As an example, using 1% (p = 0.01) resistances in Figure 2a can lower R_o from ∞ to as little as 1,000/(4×0.01) = 25 kΩ, thus making i_O depend upon v_L, by Equation 3. If the bridge is unbalanced in the opposite direction of above, then the worst-case condition for R_o is –25 kΩ. So, depending on the mismatch, R_o may lie anywhere from +25 kΩ to ∞ to –25 kΩ.

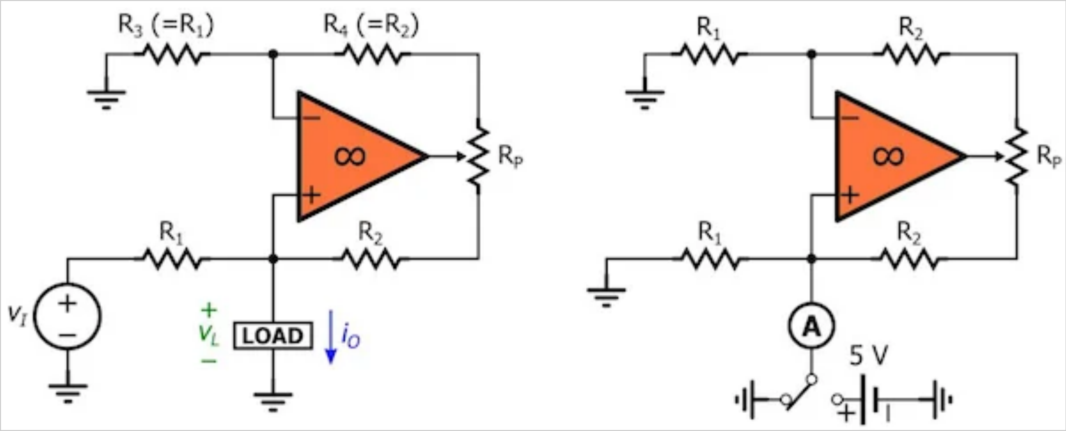

Figure 3. (a) Using a potentiometer R_p to balance the resistive bridge. (b) Calibration set up.

For improved performance, we must either use lower-tolerance resistances or balance the bridge using a potentiometer R_p, as in Figure 3a. To calibrate the circuit, ground the input as in Figure 3b and use an ammeter A. First, flip the switch to ground, and if necessary, zero the op-amp's input offset voltage until the ammeter reads zero. Then flip the switch to a known voltage, such as 5V, and adjust R_p until the ammeter reads again zero. By imposing that i_O with v_L = 5 V be equal to i_O with v_L = 0 V, we are making i_O independent of v_L, in effect driving R_o to infinity, by Equation 3.

The Effect of Op-Amp Nonidealities

Common-Mode Rejection Ratio

A practical op-amp is sensitive to its common-mode input voltage, a feature that is modeled with a small internal offset voltage in series with the noninverting input. In the case of the Howland pump, this offset voltage can be expressed as v_L/

CMRR, where CMRR is the common-mode rejection ratio as reported in the op-amp's datasheet. With reference to Figure 4a, we note that Equation 1 still holds, but Equation 2 changes to

$$v_A = \left(1+\frac{R_4}{R_3}\right)\times\left(v_L+\frac{v_L}{\mathrm{CMRR}}\right)=\left(1+\frac{R_2}{R_1}\right)\times v_L\times\left(1+\frac{1}{\mathrm{CMRR}}\right)$$

Substituting into Equation 1, solving for i_O, and putting i_O in the form of Equation 3 gives

$$R_o = (R_1 \parallel R_2)\times \mathrm{CMRR}$$

Equation 8

As an example, using an op-amp with CMRR = 60 dB (=1000) in the above example will lower R_O from ∞ to $(10^3 \| 10^3)\times 1000$ = 500 kΩ. With an arrangement of the type of Figure 3b, we can use the potentiometer to compensate for the cumulative effect of bridge imbalance as well as non-infinite CMRR.

Open-Loop Gain

So far we have assumed the op-amp to have infinite open-loop gain. The gain a of a practical op-amp is finite, so let us now see how this affects circuit behavior.

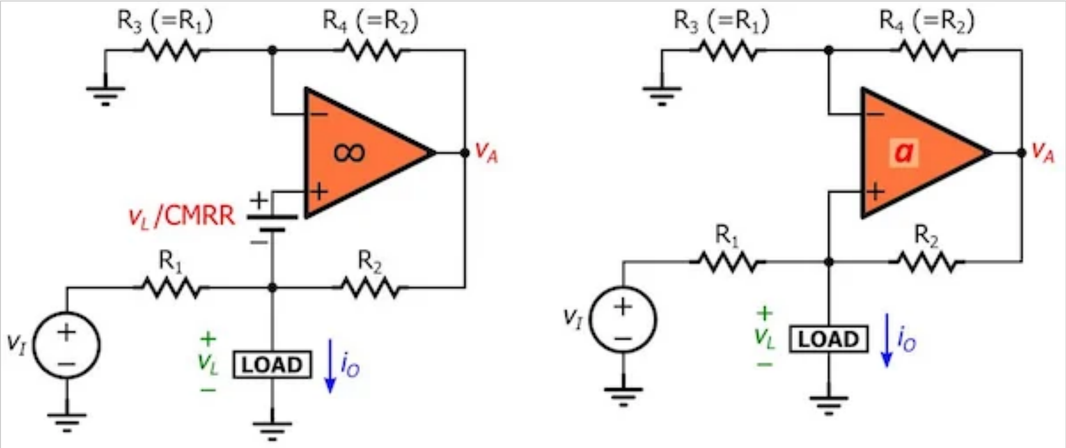

Figure 4. Circuits to investigate the effect of (a) non-infinite common-mode rejection ratio and (b) non-infinite open-loop gain.

With reference to Figure 4b, we now have

$$v_A = a\left(v_L - \frac{R_3}{R_3+R_4}v_A\right)$$

Solving for v_A, substituting into Equation 1, solving for i_O, and putting i_O in the form of Equation 3 gives

$$R_o = (R_1 \parallel R_2) \times \left(1 + \frac{a}{1 + R_2 / R_1}\right)$$

Equation 9

As an example, using an op-amp with a DC gain of 100 dB (=100,000 V/V) will lower R_o from ∞ to $(10^3 \| 10^3) \times (1 + 100{,}000/2) \cong 25$ MΩ. With an arrangement of the type of Figure 3b, we can use the potentiometer to compensate for the cumulative effect of bridge imbalance, non-infinite CMRR, and non-infinite open-loop DC gain, and raise R_o as close as possible to ∞.

However, as we increase the frequency of operation, the gain a rolls off with frequency, leading to a progressive deterioration of R_o. For example, if an op-amp with a DC gain of 100 dB has a gain-bandwidth product of 1 MHz, its open-loop gain vs. frequency (assuming a single-pole response) will look like this:

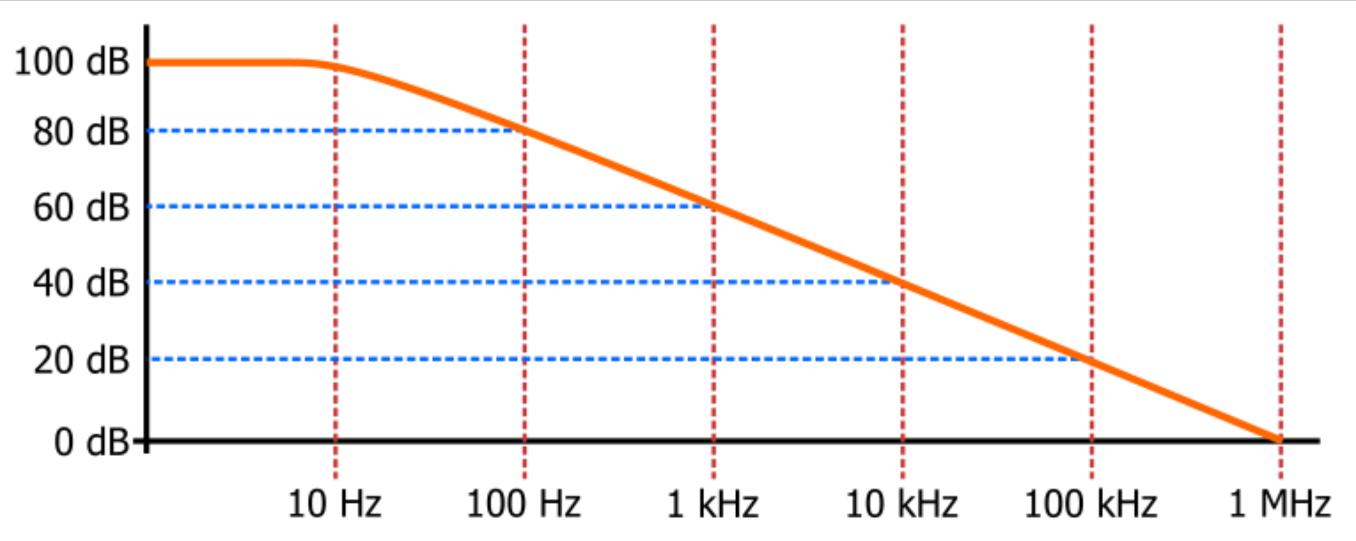

Figure 5. Single-pole frequency response of a 1 MHz op-amp with a DC open-loop gain of 100 dB.

Thus, the gain a drops to 60 dB (=1000 V/V) at 1 kHz, and the value of R_o will drop to $500 \times (1 + 1000/2) \cong 250$ kΩ. At 10 kHz R_o drops to $500 \times (1 + 100/2) \cong 25$ kΩ, and so on.

Further Reading

A Comprehensive Study of the Howland Current Pump (PDF): an application note published by Texas Instruments.

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